

GNG1106 – Fundamentals of Engineering Computation
Class Project
Mechanical Engineering

Pipe Friction – The Friction Factor

Determining fluid flow through pipes and tubes has great relevance in many areas of engineering and science. In mechanical and aerospace engineering, typical applications include the flow of liquids and gases through cooling systems.

The resistance to flow in such conduits is parameterized by a dimensionless number called the *friction factor*. For turbulent flow, the *Colebrook equation* provides a means to calculate the friction factor,

$$0 = \frac{1}{\sqrt{f}} + 2.0 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad \text{Equation 1}$$

where

f = friction factor

ε = the roughness (m),

D = conduit diameter (m),

and Re = the *Reynolds number*, defined by the following equation,

$$\text{Re} = \frac{\rho V D}{\mu} \quad \text{Equation 2}$$

where

ρ = the fluid's density [kg/m^3],

V = its velocity [m/s], and

μ = dynamic viscosity [N s/m^2].

In addition to appearing in Equation 2, the Reynolds number also serves as the criterion for whether flow is turbulent ($\text{Re} > 4000$); the Colebrook equation apply under this condition.

The various pipe parameters have values that range as follows:

Roughness ε (in millimeters): 0.0001 to 3

(e.g. cement lined cast iron pipe: 1.5, Commercial steel pipe: 0.046, drawn tubing: 0.0015)

Fluid density ρ (in kg/m^3): 0.5 (air is 1.2) to 2000 (95% sulfuric acid is 1839)

Dynamic viscosity (in N s/m^2): 10^{-6} (air is 18.6×10^{-6}) to 300 (peanut butter is 250)

Finding the value of friction factor f from Equation (1) using analytical means is far from trivial. Numerical methods can provide very good results by using a root finding method, that is, finding the root of the function $g(f)$ defined as

$$g(f) = \frac{1}{\sqrt{f}} + 2.0 \log \left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\text{Re} \sqrt{f}} \right) \quad \text{Equation 3}$$

will give the friction factor f according to the *Colebrook equation* of Equation 1.

The velocity V (m/s) to compute the Reynold's number (Re) is related to the flow rate. The flow rate, F_r , is defined as the volume of fluid that passes a location per unit time (units used is m^3/s). The relationship between the flow rate and the fluid velocity is

$$F_r = AV \quad \text{Equation 4}$$

where F_r is the flow rate in m^3/s
 A is the cross sectional area of the conduit (m^2)
 V is the fluid velocity (m/s).

The *bisection method* for finding roots, which is alternatively called binary chopping, interval halving, or Bolzano's method, is one type of incremental search method in which the interval is always divided in half. If a function changes sign over an interval, the function value at the midpoint is evaluated. The location of the root is then determined as lying at the midpoint of the subinterval within which the sign change occurs. The process is repeated to obtain refined estimates. A simple algorithm for the bisection calculation to finding the root of the function $f(x)$ consists of the following steps.

Step 1: Choose lower x_L and upper x_U guesses for the root such that the function changes sign over the interval. This can be checked by ensuring that $f(x_L)f(x_U) < 0$.

Step 2: An estimate of the root x_R is determined by $x_R = (x_L + x_U)/2$.

Step 3: Make the following evaluations to determine in which subinterval the root lies:

(a) If $f(x_L)f(x_R) < 0$, the root lies in the lower subinterval. Therefore, set $x_U = x_R$ and return to step 2.

(b) If $f(x_L)f(x_R) > 0$, the root lies in the upper subinterval. Therefore, set $x_L = x_R$ and return to step 2.

(c) If $|f(x_L)f(x_R)| < \text{some small tolerance value}$ (that is, can be considered equal to 0), the root equals x_R ; terminate the computation.

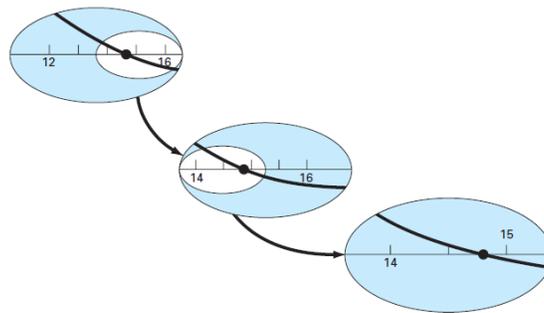


Figure 1 – Bisection Method for Finding Roots

In Figure 1, the searching for the root is started on the interval between the values of 12 and 16 where the function changes sign.

- Therefore, the initial estimate of the root x_R lies at the midpoint of the interval $x_R = (12+16)/2 = 14$.
- Next the product of the function value is computed at the lower bound and at the midpoint: $f(12)f(14) = 6.067(1.569) = 9.517$ which is greater than zero, and hence no sign change occurs between the lower bound and the midpoint. Consequently, the root must be located between 14 and 16.
- Therefore, we create a new interval by redefining the lower bound as 14.
- A revised root estimate is determined as $x_R = (14+16)/2 = 15$.

- The process can be repeated to obtain refined estimates. For example, $f(14)f(15) = 1.569(-0.425) = -0.666$. Therefore, the root is between 14 and 15.
- The upper bound is redefined as 15, and the root estimate for the third iteration is calculated as $x_R = (12+16)/2 = 14$ which represents a percent relative error of $\varepsilon = 1.9\%$.
- The method can be repeated until the result is accurate enough.

Develop software that plots how the friction factor varies with change of the conduit diameter while keeping the fluid flow rate constant. The user will provide the following conduit and fluid parameters: the pipe roughness, a minimum and maximum diameter value; and the fluid density, flow rate and viscosity. (Note: the maximum diameter value will give the minimum fluid velocity for the given flow rate; thus this fluid velocity can be used to check the Re number criterion (>4000) for validating input values).

When new input values are given, the user is given the option to save them into a file; up to five sets of values can be stored. Thus when the software starts the user can elect to use one of the five stored values or enter new values.

A bisection root finding method is used to find the friction factor.

Additional Information

An exact analytical solution for finding the friction factor is not available. The most popular solutions are numerical solutions. But there exists an approximate explicit solution called the *Swamee-Jain* equation that can provide an approximate estimate.

$$f = \frac{1.325}{\left[\ln \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2}$$

Pay attention to units. For example ε is given in mm, but must have units of meters in the above equation (as well as the Colebrook equation). The following is an example calculation:

Input values:

$$\begin{aligned} \rho &= 1.23 \text{ kg/m}^3, \\ V &= 40 \text{ m/s}, \\ \mu &= 1.79 \times 10^{-5} \text{ N s/m}^2, \\ D &= 0.005 \text{ m}, \\ \varepsilon &= 0.0015 \text{ mm} = 1.5 \times 10^{-6} \text{ m}. \end{aligned}$$

Thus

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1.23(40)0.005}{1.79 \times 10^{-5}} = 13743$$

($\text{Re} > 4000$, which meets the condition of turbulence to apply the Colebrook equation). Using the *Swamee-Jain* equation, the friction factor can be estimated as:

$$f = \frac{1.325}{\left[\ln \left(\frac{\varepsilon}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} = \frac{1.325}{\left[\ln \left(\frac{1.5 \times 10^{-6}}{3.7D} + \frac{5.74}{(13743)^{0.9}} \right) \right]^2} = 0.029$$

Note that friction factors range from about 0.008 to 0.08. Use this information to develop your test cases. Do not forget to add this information in section 2 of your project document.